

Spontaneous R-symmetry breaking in O’Raifeartaigh models

David Shih

*Department of Physics, Harvard University,
Cambridge, MA 02138 U.S.A.*

E-mail: dshih@physics.harvard.edu

ABSTRACT: We study the question of whether spontaneous $U(1)_R$ breaking can occur in O’Raifeartaigh-type models of spontaneous supersymmetry breaking. We show that in order for it to occur, there must be a field in the theory with R-charge different from 0 or 2. We construct the simplest O’Raifeartaigh model with this property, and we find that for a wide range of parameters, it has a meta-stable vacuum where $U(1)_R$ is spontaneously broken. This suggests that spontaneous $U(1)_R$ breaking actually occurs in generic O’Raifeartaigh models.

KEYWORDS: Global Symmetries, Supersymmetry Breaking.

Contents

1. Introduction	1
2. General results for O’Raifeartaigh models	3
2.1 A general formula for m_X^2	3
2.2 R-charge assignments and spontaneous $U(1)_R$ breaking	6
3. The simplest O’Raifeartaigh model with spontaneous $U(1)_R$ breaking	7
3.1 The model and its vacuum	7
3.2 Lifetime estimate	10
A. Some examples	11
A.1 Example 1: the original O’Raifeartaigh model	11
A.2 Example 2: Witten’s $SU(5)$ “inverted hierarchy” model	12
A.3 Example 3: the $SU(2)$ “rank condition” model	13

1. Introduction

Recently, there has been a revival of interest in low-scale SUSY model building using renormalizable, perturbative models of spontaneous supersymmetry breaking — i.e. generalizations of the O’Raifeartaigh model [1] — in the hidden sector [2–10]. This has been motivated in part by the realization that O’Raifeartaigh-type models can arise naturally and dynamically in the low-energy limit of simple SUSY gauge theories such as massive SQCD [11].

In all of the recent model building attempts, one common theme has been the R-symmetry. According to [12], this must exist in any generic, calculable theory of spontaneous F-term supersymmetry breaking; but at the same time, it must be broken in order to have nonzero Majorana gaugino masses. Because the vacuum of the simplest O’Raifeartaigh models preserves the R-symmetry (for a recent review of this and other facts about O’Raifeartaigh models, see e.g. [13]), the models built to date have focused on two mechanisms for breaking the R-symmetry, both of which involve modifying the O’Raifeartaigh model in some way. These are: adding explicit R-symmetry violating operators to the superpotential; or gauging a flavor symmetry and using gauge interactions to spontaneously break the R-symmetry. Neither of these mechanisms for R-symmetry breaking are completely free of problems. Explicitly breaking the R-symmetry tends to restore supersymmetry, and this can sometimes lead to tension between having a sufficiently long-lived meta-stable vacuum and having sufficiently large gaugino masses. (This is not always a serious problem; see e.g. the recent model of [5].) Meanwhile, using gauge interactions to spontaneously break R-symmetry typically leads to an “inverted hierarchy” [14], which

is problematic for models of low-scale SUSY breaking. One can achieve spontaneous R-symmetry breaking without the inverted hierarchy (see for instance the early model of [15], and more recently [3, 6]), but this seems to generally require significant fine tuning of the couplings [16].

In this paper, we propose to consider a much simpler mechanism for R-symmetry breaking, namely using the perturbative dynamics of the O’Raifeartaigh model itself to spontaneously break $U(1)_R$.

To see how this could come about, recall that in O’Raifeartaigh-type models, there is always a pseudo-moduli space, i.e. a continuous space of supersymmetry-breaking vacua with degenerate tree-level vacuum energies [17]. For simplicity, we will focus on models with a single pseudo-modulus X . Expanding around the pseudo-moduli space, the superpotential of such models can always be put in the form

$$W = fX + \frac{1}{2} (M^{ij} + XN^{ij}) \phi_i \phi_j + \dots \tag{1.1}$$

so that the pseudo-moduli space occurs at $\phi_i = 0$, with X arbitrary. Here X and the ϕ_i are chiral superfields, and \dots denote possible cubic interactions amongst the ϕ_i fields. These are irrelevant for the calculation of the one-loop Coleman-Weinberg potential, which depends only on the mass matrices of the ϕ_i superfields evaluated at $\phi_i = 0$ (assumed to be positive definite in a neighborhood around $X = 0$):

$$V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2} \text{Tr} (-1)^F \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \tag{1.2}$$

The R-symmetry present in (1.1) implies that $R(X) = 2$, and guarantees that to leading order around $X = 0$, the effective potential takes the form

$$V_{\text{eff}}^{(1)} = V_0 + m_X^2 |X|^2 + \mathcal{O}(|X|^4) \tag{1.3}$$

The sign of m_X^2 then determines whether or not the R-symmetry is spontaneously broken.

In section 2, we derive a general formula for m_X^2 in terms of the matrices M and N appearing in (1.1). We observe that m_X^2 can in general have either sign, and we show that a *necessary* condition for m_X^2 to be negative is that there exists a field ϕ_i in (1.1) with R-charge other than 0 or 2.

In section 3, we construct the simplest O’Raifeartaigh model with this property: a model with chiral fields $\phi_{1,2,3}$ having R-charges $-1, 1$ and 3 respectively, and superpotential

$$W = \lambda X \phi_1 \phi_2 + m_1 \phi_1 \phi_3 + \frac{1}{2} m_2 \phi_2^2 + fX \tag{1.4}$$

We show that for a wide range of parameters, $m_X^2 < 0$ and there is a local minimum of the potential with $X \neq 0$ and spontaneously broken R-symmetry. Interestingly, this model has runaway behavior at large fields, so the vacuum we find at $X \neq 0$ is only meta-stable. However, it can be made parametrically long-lived in the limit $y \equiv |\lambda f / m_1 m_2| \rightarrow 0$.

Finally, in the appendix, we provide a few consistency checks of our general formula for m_X^2 . We apply our general formula to a few well-known examples which we believe are

representative of the O’Raifeartaigh models used so far in model building. This includes the original O’Raifeartaigh model [1],

$$W = fX + m\phi_1\phi_2 + \frac{1}{2}hX\phi_1^2 \tag{1.5}$$

as well as Witten’s SU(5) “inverted hierarchy” model [14], and the simplest version of the “rank condition” models of [11]. In all of these models, all of the fields have either $R = 0$ or $R = 2$, and consequently, the R-symmetry remains unbroken.

The fact that spontaneous R-symmetry breaking already occurs (for some range of the couplings) in the simplest model with more general R-charge assignments suggests that it actually occurs (again, for some range of the couplings) in a *generic* O’Raifeartaigh-type model. One reason this may have gone unnoticed until now is that many, if not all, of the O’Raifeartaigh models considered to date (such as the ones in the appendix) share the highly non-generic R-charge assignments ($R = 0$ or $R = 2$) of the original model (1.5).

The possibility of spontaneous R-symmetry breaking in O’Raifeartaigh models opens up many new directions which would be interesting to explore. For instance, it would be useful to find more examples of O’Raifeartaigh models with spontaneous R-symmetry breaking, especially ones with larger global symmetries. One interesting question is whether the runaway behavior seen in the example (1.4) is a general feature of these examples. Also, it would be interesting to explore the “retro-fitting” of these models along the lines of [2], and to search for simple, asymptotically-free UV completions along the lines of [11]. Finally, the application of these ideas to phenomenology is a promising direction which could potentially lead to new models of supersymmetry breaking, especially models of low-scale direct mediation.

2. General results for O’Raifeartaigh models

2.1 A general formula for m_X^2

Let us start by defining our class of models (1.1) more precisely. As described in the introduction, we will consider the most general O’Raifeartaigh type model with a single pseudo-modulus X and an R symmetry. Thus, we have a renormalizable Wess-Zumino model consisting of a chiral superfield X and n chiral superfields ϕ_i , with canonical Kähler potential and superpotential

$$W = fX + \frac{1}{2}(M^{ij} + XN^{ij})\phi_i\phi_j \tag{2.1}$$

Here M and N are symmetric complex matrices, and we will assume that $\det M \neq 0$. We will take f to be real and positive without loss of generality. The R-symmetry implies that $R(X) = 2$, and it constrains the form of M and N :

$$M^{ij} \neq 0 \Rightarrow R(\phi_i) + R(\phi_j) = 2; \quad N^{ij} \neq 0 \Rightarrow R(\phi_i) + R(\phi_j) = 0 \tag{2.2}$$

Combining this with $\det M \neq 0$, we see that, in a basis where fields of the same R-charge are grouped together, M must have the block form

$$M = \begin{pmatrix} & & & & M_1 \\ & & & & \\ & & & M_2 & \\ & & & \cdot & \\ & & & \cdot & \\ & & & \cdot & \\ & & & \cdot & \\ M_1^T & M_2^T & & & \end{pmatrix} \quad (2.3)$$

where the M_i are individually square, non-degenerate matrices. One consequence of this (which we will need in the next paragraph) is that M^{-1} has the same block form as M and so it also satisfies the same relations (2.2) as M .

Another important consequence of the R-symmetry is that it implies that supersymmetry is broken. We can prove this with a direct computation of $\det(M + XN)$. Supersymmetry is broken if this quantity is nonzero and independent of X , since then the ϕ_i F-terms and the X F-term are incompatible. We find:

$$\begin{aligned} \det(M + XN) &= \exp(\text{Tr} \log(\mathbf{1}_n + XM^{-1}N)) \det M \\ &= \exp\left(-\sum_{k \geq 1} \frac{(-X)^k}{k} \text{Tr}(M^{-1}N)^k\right) \det M \end{aligned} \quad (2.4)$$

By the R-symmetry relations (2.2), $(M^{-1}N)_i^j$ is nonzero only if $R(\phi_i) - R(\phi_j) = 2$. Thus any power of $M^{-1}N$ will have vanishing diagonal entries, and so the traces in (2.4) all vanish. Therefore,

$$\det(M + XN) = \det M \neq 0 \quad (2.5)$$

and supersymmetry is broken. Note that this is a stronger statement than that of [12], which argues that an R-symmetry is only a *necessary* condition for spontaneous F-term supersymmetry breaking in a generic WZ model. Here we have seen that for O’Raifeartaigh models (2.1), the R-symmetry is also a *sufficient* condition for supersymmetry breaking. The point is that the models (2.1) are not completely generic; in particular, all the terms are at most linear in X . (They can be made “generic” if we also impose an obvious \mathbb{Z}_2 symmetry in addition to the R-symmetry.)

The scalar potential of this model has a one-dimensional space of extrema given by

$$\phi_i = 0, \quad X \text{ arbitrary}, \quad V_0 = f^2 \quad (2.6)$$

This may or may not be the absolute minimum of the tree-level potential, depending on the details of M and N . In particular, there could be other, lower energy pseudo-moduli spaces with $\phi_i \neq 0$, and there could also be runaway behavior at large fields. However, we will assume that the couplings are such that (2.6) is at least a local minimum of the potential in a neighborhood around $X = 0$.

The R-symmetry implies that the effective potential V_{eff} on the pseudo-moduli space has an extremum at $X = 0$:

$$V_{\text{eff}} = V_{\text{eff}}(|X|^2) = \text{const.} + m_X^2 |X|^2 + \mathcal{O}(|X|^4) \quad (2.7)$$

Our goal in this subsection is to derive a general formula for m_X^2 in the one-loop approximation. We will do this by expanding the usual Coleman-Weinberg formula

$$V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2} \text{Tr} (-1)^F \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \quad (2.8)$$

to quadratic order in X . Here \mathcal{M}^2 is shorthand for \mathcal{M}_B^2 and \mathcal{M}_F^2 , the mass matrices of the scalar and fermion components of the superfields ϕ_i , respectively:

$$\begin{aligned} \mathcal{M}_B^2 &= \begin{pmatrix} W_{ik}^\dagger W^{kj} & W_{ijk}^\dagger W^k \\ W^{ijk} W_k^\dagger & W^{ik} W_{kj}^\dagger \end{pmatrix} = \left(\widehat{M} + X \widehat{N} \right)^2 + f \widehat{N} \\ \mathcal{M}_F^2 &= \begin{pmatrix} W_{ik}^\dagger W^{kj} & 0 \\ 0 & W^{ik} W_{kj}^\dagger \end{pmatrix} = \left(\widehat{M} + X \widehat{N} \right)^2 \end{aligned} \quad (2.9)$$

where $W^i \equiv \partial W / \partial \phi_i$, etc., and we have defined

$$\widehat{M} \equiv \begin{pmatrix} 0 & M^\dagger \\ M & 0 \end{pmatrix}, \quad \widehat{N} \equiv \begin{pmatrix} 0 & N^\dagger \\ N & 0 \end{pmatrix} \quad (2.10)$$

Note that we are taking X to be real, which suffices for extracting m_X^2 , according to (2.7).

In order to expand (2.8) in X , it helps to rewrite it in the following form:

$$V_{\text{eff}}^{(1)} = -\frac{1}{32\pi^2} \text{Tr} \int_0^\Lambda dv v^5 \left(\frac{1}{v^2 + \mathcal{M}_B^2} - \frac{1}{v^2 + \mathcal{M}_F^2} \right) \quad (2.11)$$

Substituting (2.9) into (2.11) and expanding to order X^2 , we obtain (after an integration by parts)

$$\begin{aligned} m_X^2 &= \frac{1}{16\pi^2} \text{Tr} \int_0^\Lambda dv v^3 \left[\frac{1}{v^2 + \widehat{M}^2 + f \widehat{N}} \left(\widehat{N}^2 - \frac{1}{2} \{ \widehat{M}, \widehat{N} \} \frac{1}{v^2 + \widehat{M}^2 + f \widehat{N}} \{ \widehat{M}, \widehat{N} \} \right) \right. \\ &\quad \left. - \frac{1}{v^2 + \widehat{M}^2} \left(\widehat{N}^2 - \frac{1}{2} \{ \widehat{M}, \widehat{N} \} \frac{1}{v^2 + \widehat{M}^2} \{ \widehat{M}, \widehat{N} \} \right) \right] \end{aligned} \quad (2.12)$$

We can simplify this formula by expressing it in terms of

$$\mathcal{F}(v) \equiv \left(v^2 + \widehat{M}^2 \right)^{-1} f \widehat{N} \quad (2.13)$$

After using (2.2) to eliminate some of the resulting terms, we arrive at

$$m_X^2 = \frac{1}{16\pi^2 f^2} \int_0^\infty dv v^3 \text{Tr} \left[\frac{\mathcal{F}(v)^4}{1 - \mathcal{F}(v)^2} v^2 - 2 \left(\frac{\mathcal{F}(v)^2}{1 - \mathcal{F}(v)^2} \widehat{M} \right)^2 \right] \quad (2.14)$$

This is our final expression for the mass-squared of X around the origin. In the appendix, we will provide some consistency checks of (2.14) by applying it to some well-studied examples. Some comments on this result:

1. Except in the simplest models, this formula generally provides a more efficient means of computing m_X^2 , compared with first diagonalizing the mass matrices, computing the Coleman-Weinberg potential, and then expanding in X .

2. (2.14) is a difference of two non-negative quantities,

$$m_X^2 = M_1^2 - M_2^2 \quad (2.15)$$

where

$$\begin{aligned} M_1^2 &= \frac{1}{16\pi^2 f^2} \int_0^\infty dv v^5 \text{Tr} \frac{\mathcal{F}(v)^4}{1 - \mathcal{F}(v)^2} \\ M_2^2 &= \frac{1}{8\pi^2 f^2} \int_0^\infty dv v^3 \text{Tr} \left[\left(\frac{\mathcal{F}(v)^2}{1 - \mathcal{F}(v)^2} \widehat{M} \right)^2 \right] \end{aligned} \quad (2.16)$$

In the absence of any inequalities relating these quantities, m_X^2 will be of indefinite sign. This raises the possibility of spontaneous R-breaking in an O’Raifeartaigh type model, without the need for gauge interactions. In the next subsection, we will derive a necessary condition which must be satisfied in order for this to occur.

2.2 R-charge assignments and spontaneous $U(1)_R$ breaking

We claim that in O’Raifeartaigh models where all fields have R-charge either 0 or 2, $M_2^2 = 0$ and $m_X^2 = M_1^2 > 0$. Thus, in order for m_X^2 to be negative, there must be at least one field in the theory with R-charge other than 0 or 2.¹

The proof is straightforward. If all of the ϕ_i have R-charge either 0 or 2, then according to (2.2), N and M must take the form

$$M = \begin{pmatrix} 0 & M_{02} \\ M_{02}^T & 0 \end{pmatrix}, \quad N = \begin{pmatrix} N_{00} & 0 \\ 0 & 0 \end{pmatrix} \quad (2.17)$$

in a basis where the fields with R-charge 0 and 2 are grouped together into blocks. Hence,

$$\widehat{M} = \begin{pmatrix} 0 & 0 & 0 & M_{02}^* \\ 0 & 0 & M_{02}^\dagger & 0 \\ 0 & M_{02} & 0 & 0 \\ M_{02}^T & 0 & 0 & 0 \end{pmatrix}, \quad \widehat{N} = \begin{pmatrix} 0 & 0 & N_{00}^\dagger & 0 \\ 0 & 0 & 0 & 0 \\ N_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.18)$$

Substituting into (2.13), we find that \mathcal{F} has the same block form as \widehat{N} :

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & \mathcal{F}_{00}^\dagger & 0 \\ 0 & 0 & 0 & 0 \\ \mathcal{F}_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.19)$$

Finally, substituting into (2.16) gives

$$\left(\frac{\mathcal{F}^2}{1 - \mathcal{F}^2} \widehat{M} \right)^2 = \begin{pmatrix} 0 & 0 & 0 & \frac{\mathcal{F}_{00}^\dagger \mathcal{F}_{00}}{1 - \mathcal{F}_{00}^\dagger \mathcal{F}_{00}} M_{02}^* \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\mathcal{F}_{00} \mathcal{F}_{00}^\dagger}{1 - \mathcal{F}_{00} \mathcal{F}_{00}^\dagger} M_{02} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^2 = 0 \quad (2.20)$$

¹Note that if the theory has other global $U(1)$ symmetries, then any combination of these with the R-symmetry is also an R-symmetry. So a more precise statement of our claim is: if there is some choice of the R-symmetry such that all the fields have $R = 0$ or $R = 2$, then $m_X^2 = M_1^2 > 0$.

so M_2^2 vanishes. Meanwhile,

$$\text{Tr} \frac{\mathcal{F}^4}{1 - \mathcal{F}^2} = 2 \text{Tr} \frac{(\mathcal{F}_{00}^\dagger \mathcal{F}_{00})^2}{1 - \mathcal{F}_{00}^\dagger \mathcal{F}_{00}} > 0 \tag{2.21}$$

so $m_X^2 = M_1^2$ is non-vanishing and positive. This completes the proof of the claim.

The class of O’Raifeartaigh models where all fields have $R = 0$ or $R = 2$ may seem very non-generic, but it actually characterizes many (if not all!) of the O’Raifeartaigh models studied in the literature. In all of these models, the R-symmetry was found to be unbroken in the vacuum, and now we see that this is a direct consequence of the R-charge assignments. (An earlier hint of this came from the work of [18], who showed that the R-symmetry is unbroken in a particular subset of models of this type.) The examples in the appendix will illustrate this point in detail.

3. The simplest O’Raifeartaigh model with spontaneous $U(1)_R$ breaking

3.1 The model and its vacuum

We have seen that a necessary condition for spontaneous R-symmetry breaking in O’Raifeartaigh models is that there is a field with R-charge other than 0 or 2. In this section we will construct and analyze the simplest O’Raifeartaigh model of this type. We will see that, indeed, spontaneous R-symmetry breaking occurs in this model for a wide range of the parameters.

Our “simplest model” is constructed in the following way. Suppose we have a field ϕ_1 , in addition to X , with $R(\phi_1) = n$. This field must have a mass term, so there must be another field ϕ_3 with $R(\phi_3) = 2 - n$. In addition, it should feel the SUSY breaking, so there must be a field ϕ_2 with $R(\phi_2) = -n$ to allow for the coupling $X\phi_1\phi_2$. Finally, ϕ_2 needs a mass term, so there must be a field ϕ_4 with $R(\phi_4) = 2 + n$. Thus we need at least four fields in addition to X — except in the degenerate cases $n = 0$ and $n = \pm 1$. When $n = 0$, ϕ_1 and ϕ_2 can be identified, as can ϕ_3 and ϕ_4 . This is the original O’Raifeartaigh model. In the cases $n = -1$ or $n = 1$, ϕ_4 or ϕ_3 are redundant, respectively. The two are equivalent after a trivial field redefinition, so we will take $n = -1$ without loss of generality. The most general renormalizable superpotential consistent with the R-symmetry is:

$$W = \lambda X \phi_1 \phi_2 + m_1 \phi_1 \phi_3 + \frac{1}{2} m_2 \phi_2^2 + f X \tag{3.1}$$

This is the simplest O’Raifeartaigh-type model containing a field with $R \neq 0, 2$.

The scalar potential is

$$V = |\lambda \phi_1 \phi_2 + f|^2 + |\lambda X \phi_2 + m_1 \phi_3|^2 + |\lambda X \phi_1 + m_2 \phi_2|^2 + |m_1 \phi_1|^2 \tag{3.2}$$

By rotating the phases of all the fields, we can always take all the couplings to be real and positive, without loss of generality. The extrema of the potential consist of a pseudo-moduli space

$$\phi_i = 0, \quad X \text{ arbitrary} \tag{3.3}$$

In addition, there is runaway behavior as $\phi_3 \rightarrow \infty$

$$X = \left(\frac{m_1^2 m_2 \phi_3^2}{\lambda^2 f} \right)^{1/3}, \quad \phi_1 = \left(\frac{f^2 m_2}{\lambda^2 m_1 \phi_3} \right)^{1/3}, \quad \phi_2 = \left(\frac{f m_1 \phi_3}{\lambda m_2} \right)^{1/3}, \quad \phi_3 \rightarrow \infty \quad (3.4)$$

As a check, note that the scaling exhibited in (3.4) is consistent with the R-symmetry.

The runaway behavior at large fields implies that the pseudo-moduli space is not an absolute minimum of the potential. However, as long as

$$|X| < \frac{m_1}{\lambda} \frac{1 - y^2}{2y} \quad (3.5)$$

the pseudo-moduli space is a local minimum of the potential. Here we have defined

$$y = \frac{\lambda f}{m_1 m_2} \quad (3.6)$$

For $|X|$ larger than the bound, a linear combination of the ϕ_i fields becomes tachyonic, and the system can roll down classically into a runaway direction.

Note that for $y > 1$, the pseudo-moduli space is unstable for all X , while for $y < 1$ there is some neighborhood around $X = 0$ which is stable. The size of this neighborhood grows monotonically as $y \rightarrow 0$.

Now let us compute the mass-squared of X around the origin. In this simple example, it is straightforward to compute directly the full Coleman-Weinberg potential, and then expand around the origin to extract m_X^2 . However, let us compute it using the general formula (2.14), so that we can see the effect of having a field with $R \neq 0, 2$. The matrices M and N in (2.1) are

$$M = \begin{pmatrix} 0 & 0 & m_1 \\ 0 & m_2 & 0 \\ m_1 & 0 & 0 \end{pmatrix}, \quad N = \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.7)$$

Substituting into (2.13) and (2.14), we find that both terms of (2.14) are now nonzero — a consequence of the fact that there are fields with $R \neq 0, 2$ in the model. The expressions are quite complicated, but they simplify in the small y limit:

$$M_1^2 = \frac{m_1^2 \lambda^2 y^2}{8\pi^2} \frac{r^2 (-4r^2 \log r + r^4 - 1)}{(r^2 - 1)^3} + \mathcal{O}(y^4)$$

$$M_2^2 = \frac{m_1^2 \lambda^2 y^2}{4\pi^2} \frac{r^4 ((r^2 + 1) \log r - r^2 + 1)}{(r^2 - 1)^3} + \mathcal{O}(y^4) \quad (3.8)$$

where we have defined $r = m_2/m_1$. Their difference is

$$m_X^2 = M_1^2 - M_2^2 = -\frac{m_1^2 \lambda^2 y^2}{8\pi^2} \frac{r^2 (2r^2(r^2+3) \log r - (3r^4 - 2r^2 - 1))}{(r^2 - 1)^3} + \mathcal{O}(y^4) \quad (3.9)$$

A plot of the function of r appearing in (3.9) is shown in figure 1. We see that for small r , m_X^2 is positive, while for large r it is negative. The turnover point is at

$$r = r_* \approx 2.11 \quad (3.10)$$

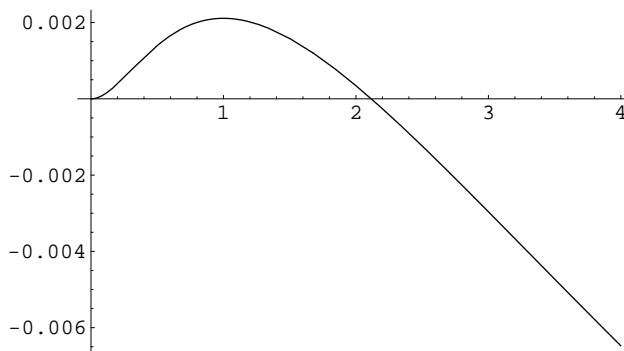


Figure 1: A plot of m_X^2 vs. $r = m_2/m_1$ in the small $y = \lambda f/m_1 m_2$ limit. We see that for $r > r_* \approx 2.11$, the mass-squared of X around the origin is negative, and the R-symmetry is spontaneously broken.

This is valid in the small y limit; more generally, r_* is a function of y .

We have shown that for $r > r_*$, the mass-squared of X is negative around the origin, but we still need to check that there is a local minimum with $X \neq 0$. Let us check this analytically, again in the small y limit. Expanding the effective potential to $\mathcal{O}(|X|^4)$, we find

$$V_{\text{eff}}^{(1)} = \text{const.} + m_X^2 |X|^2 + \frac{1}{4} \lambda_X |X|^4 + \mathcal{O}(|X|^6) \quad (3.11)$$

where

$$\lambda_X = \frac{3\lambda^4 y^2 r^2 (12r^2(r^4 + 5r^2 + 2) \log r - 19r^6 - 9r^4 + 27r^2 + 1)}{8\pi^2 (r^2 - 1)^5} + \mathcal{O}(y^4) \quad (3.12)$$

For $r \geq r_*$, $m_X^2 \leq 0$, but λ_X is strictly positive. Balancing the quadratic and quartic terms, we find a local minimum at

$$|X|^2 \approx \frac{2|m_X^2|}{\lambda_X} = \frac{m_1^2}{\lambda^2} f(r) \quad (3.13)$$

Since $f(r)$ is a monotonically increasing and unbounded function of $r \geq r_*$, satisfying $f(r_*) = 0$, this approximation is valid for some range of r above r_* , where $|X|$ is sufficiently small that the expansion (3.11) can be trusted. Note that (3.13) implies that $|X|$ is $\mathcal{O}(y^0)$ in the $y \rightarrow 0$ limit, so by taking y parametrically small, we can always satisfy the tachyon-free bound (3.5). We conclude that, at least for infinitesimal y and some range of r above r_* , and there is a local minimum of the potential at $|X| \neq 0$.

For more general values of the parameters, the analytic approach to minimizing the potential becomes intractable. However, for a given set of couplings (r, y) , it is straightforward to numerically minimize the full one-loop Coleman-Weinberg potential. One can then scan over a grid of couplings and determine the region in coupling space where the minimum of the potential breaks $U(1)_R$ and satisfies (3.5). The result of such a numerical analysis is shown in figure 2.

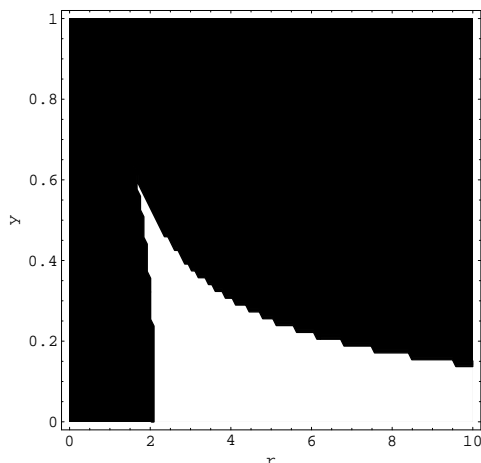


Figure 2: A plot of the region (shown in white) in the r, y plane where there is a $U(1)_R$ -breaking local minimum of the potential satisfying (3.5).

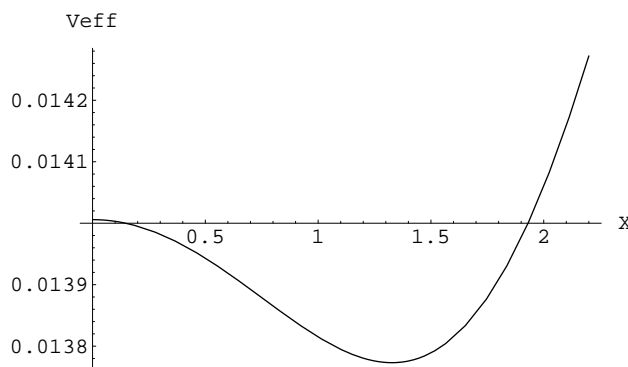


Figure 3: A plot of $V_{\text{eff}}^{(1)}$ vs. $|X|$ for the theory (3.1) with $m_1 = 1, m_2 = 4, \lambda = 1, y = 0.2$. The R-breaking local minimum of the potential occurs at $|X| \approx 1.3$. According to the bound (3.5), a transverse direction (a linear combination of the ϕ_i fields) becomes tachyonic for $|X| > 2.4$.

To summarize, we find that for a wide range of parameters there is a local $U(1)_R$ -breaking minimum of the potential at $X \neq 0$ and satisfying (3.5). A plot of the full one-loop effective potential is shown in figure 3 for a representative choice of the parameters. In general, $X \sim \mathcal{O}(m_*)$ where m_* is some characteristic mass scale of the model determined by m_1, m_2 and f . Therefore, this is spontaneous $U(1)_R$ -breaking without inverted hierarchy. This could be useful for low-scale model building, especially since the more conventional approach of gauging a flavor symmetry seems to lead to a non-hierarchical $U(1)_R$ -breaking phase only in a relatively narrow window of coupling space [16].

3.2 Lifetime estimate

Finally, let us briefly discuss the lifetime of the R-symmetry breaking vacuum found in the

previous subsection. This vacuum is only meta-stable, because of the runaway directions at large fields (3.4). However, we expect that the lifetime of the vacuum is controlled by the parameter y , and in the small y limit it is parametrically long-lived. We can estimate the lifetime by noting that along the runaway direction (3.4), the value of $|X|$ at which the potential energy becomes equal to the false vacuum energy $|f|^2$ is

$$|X| = \frac{m_1}{\lambda} y^{-1} \tag{3.14}$$

For smaller values of $|X|$, the potential energy is larger than f^2 along the runaway direction. So this indicates that the barrier width scales like y^{-1} (since the meta-stable vacuum is at $X \sim \mathcal{O}(y^0)$ according to (3.13)). Since the barrier height is $\mathcal{O}(y^0)$, this is enough to guarantee that the meta-stable vacuum is parametrically long-lived in the $y \rightarrow 0$ limit.

Acknowledgments

We would like to thank Michael Dine, Ken Intriligator, Patrick Meade and Nathan Seiberg for useful discussions and comments on the draft. This work has been supported in part by the DOE grant DE-FG0291ER40654.

A. Some examples

In this appendix, we will apply our general formula for m_X^2 (2.14) to some of the most well-studied O’Raifeartaigh-type models, which all happen to have the property that all fields have either $R = 0$ or $R = 2$. This will serve as a consistency check of the above calculations. In all of the examples, we will assume without loss of generality that all the couplings are real and positive.

A.1 Example 1: the original O’Raifeartaigh model

The first example is the basic O’Raifeartaigh model. The simplest generalizations of this model were featured in some of the earliest SUSY model building attempts [19–21, 15]:

$$W = \frac{1}{2} h X \phi_1^2 + m \phi_1 \phi_2 + f X \tag{A.1}$$

Here $R(X) = R(\phi_2) = 2$ and $R(\phi_1) = 0$. When $y \equiv \frac{hf}{m^2} < 1$, the pseudo-moduli space is (2.6), with $M = m \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $N = h \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. From this, we find

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & \frac{hf}{v^2+m^2} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{hf}{v^2+m^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.2}$$

Substituting into (2.14), we see that the second term is zero, while the first term is nonzero, yielding

$$\begin{aligned}
 m_X^2 &= \frac{1}{16\pi^2 f^2} \int_0^\infty dv \frac{2f^4 h^4 v^5}{(v^2 + m^2)^2 ((v^2 + m^2)^2 - h^2 f^2)} \\
 &= \frac{h^2 m^2}{32\pi^2} y^{-1} \left((1+y)^2 \log(1+y) - (1-y)^2 \log(1-y) - 2y \right) \quad (\text{A.3})
 \end{aligned}$$

which is the expected (positive) result.

What about when $y > 1$? Although the potential is minimized along a pseudo-moduli space

$$\phi_2 = -\frac{hX}{m}\phi_1, \quad \phi_1 = \pm \frac{im}{h} \sqrt{2(y-1)}, \quad X \text{ arbitrary} \quad (\text{A.4})$$

which is not of the form (2.6), the general formula (2.14) for m_X^2 can still be applied after performing a unitary transformation and constant shift on the fields. Specifically, we take

$$\phi_1 = \tilde{\phi}_1 \pm \frac{im}{h} \sqrt{2(y-1)}, \quad \begin{pmatrix} \phi_2 \\ X \end{pmatrix} = \begin{pmatrix} \cos \theta & \mp i \sin \theta \\ \mp i \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \tilde{\phi}_2 \\ \tilde{X} \end{pmatrix}$$

with $\cos \theta = \frac{1}{\sqrt{2y-1}}$, and this yields a superpotential of the form

$$W = \frac{1}{2} \tilde{h} \tilde{X} \tilde{\phi}_1^2 + \tilde{m} \tilde{\phi}_1 \tilde{\phi}_2 + \tilde{f} \tilde{X} + \frac{1}{2} \tilde{\lambda} \tilde{\phi}_1^2 \tilde{\phi}_2 \quad (\text{A.5})$$

where

$$\tilde{h} = \frac{h}{\sqrt{2y-1}}, \quad \tilde{m} = m\sqrt{2y-1}, \quad \tilde{f} = f \frac{\sqrt{2y-1}}{y}, \quad \tilde{\lambda} = \mp \frac{ih\sqrt{2(y-1)}}{\sqrt{2y-1}} \quad (\text{A.6})$$

Aside from the extra cubic term, this model is again of the O’Raifeartaigh form (A.1), and it has $\tilde{y} = \frac{1}{2y-1} < 1$. The cubic term does not affect the mass matrices around the pseudo-moduli space $\tilde{\phi}_i = 0$, and so the calculation of the one-loop effective potential is unchanged. Thus, the general formula (2.14) still applies, and it yields (A.3) with the couplings replaced by (A.6).

A.2 Example 2: Witten’s SU(5) “inverted hierarchy” model

This model has also featured in many early model building attempts [14, 22–24]

$$W = \frac{1}{2} h X \text{Tr} A^2 + \frac{1}{2} \lambda \text{Tr} A^2 B + f X \quad (\text{A.7})$$

where the original motivation was to explain dynamically the hierarchy between the weak scale and the GUT scale. Here A and B are adjoints of an SU(5) global symmetry, which in the original models was gauged and identified with the GUT group. Supersymmetry is broken because the X and B equations of motion are inconsistent [14]. The R-charge assignments are $R(X) = R(B) = 2$ and $R(A) = 0$.

This model has a one-dimensional pseudo-moduli space (up to global symmetries) given by

$$A = \pm i \sqrt{\frac{2fh}{30h^2 + \lambda^2}} \text{diag}(2, 2, 2, -3, -3), \quad B = \frac{hX}{\lambda} \text{diag}(2, 2, 2, -3, -3), \quad X \text{ arbitrary} \quad (\text{A.8})$$

which spontaneously breaks $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. As in the $y > 1$ phase of the previous example, we can put this in the form (2.6) by expanding around this pseudo-moduli space with a unitary transformation and constant shift:

$$A = \left(\pm i \sqrt{\frac{2fh}{30h^2 + \lambda^2}} + \frac{a}{\sqrt{30}} \right) \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} + \begin{pmatrix} A_{33} & A_{32} \\ A_{23} & A_{22} \end{pmatrix}$$

$$B = \frac{1}{\sqrt{30h^2 + \lambda^2}} \left(h\tilde{X} + \frac{\lambda b}{\sqrt{30}} \right) \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} + \begin{pmatrix} B_{33} & B_{32} \\ B_{23} & B_{22} \end{pmatrix} \quad X = \frac{1}{\sqrt{30h^2 + \lambda^2}} \left(\lambda\tilde{X} - \sqrt{30}hb \right)$$

Here A_{22} , B_{22} and A_{33} , B_{33} are adjoints of $SU(2)$ and $SU(3)$ respectively; and A_{23} , A_{32} , B_{23} , B_{32} are bifundamentals of $SU(2) \times SU(3)$. With this transformation, the superpotential becomes

$$W = \tilde{f}\tilde{X} + \frac{1}{2}\tilde{h}\tilde{X}\text{Tr}(3A_{33}^2 + A_{32}A_{23} - 2A_{22}^2) + \tilde{m}\text{Tr}(4A_{33}B_{33} - A_{32}B_{23} - A_{23}B_{32} - 6A_{22}B_{22}) + \tilde{M}ab + (\text{cubic}) \quad (\text{A.9})$$

with

$$\tilde{f} = \frac{\lambda f}{\sqrt{30h^2 + \lambda^2}}, \quad \tilde{h} = \frac{\lambda h}{\sqrt{30h^2 + \lambda^2}}, \quad \tilde{m} = \pm \frac{i\lambda}{\sqrt{2}} \sqrt{\frac{fh}{30h^2 + \lambda^2}}, \quad \tilde{M} = \mp i \sqrt{2fh} \quad (\text{A.10})$$

From this, it is straightforward to compute the matrices M and N , substitute into (2.13) and (2.14), and integrate. (Note that although there are Goldstone bosons in the spectrum because of the spontaneously broken $SU(5)$, the matrix M is still non-degenerate.) We again find that the second term of (2.14) is zero, while the first term is not. The final result is:

$$m_X^2 = \frac{3fh^3\lambda^4(968 \log 11 - 288 \log 9 + 600 \log 5 - 3038 \log 2 - 529)}{32\pi^2(30h^2 + \lambda^2)^2} > 0 \quad (\text{A.11})$$

A.3 Example 3: the $SU(2)$ “rank condition” model

For our final example, let us study the simplest “rank condition” model of [11], namely the one with $SU(2)$ global symmetry. The $SU(N_f)$ generalizations of this model have featured in many of the recent efforts at O’Raifeartaigh model building referred to in the introduction. The superpotential is

$$W = h\text{Tr}\Phi\varphi\tilde{\varphi} - h\mu^2\text{Tr}\Phi \quad (\text{A.12})$$

where Φ is an $\mathbf{adj} \oplus \mathbf{1}$ and $\varphi, \tilde{\varphi}$ are doublets under the $SU(2)$ global symmetry. This model has an R-symmetry where $R(\Phi) = 2$ and $R(\varphi) = 0$. The absolute minimum of the tree-level

scalar potential occurs along the pseudo-moduli space (modulo global symmetries)

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix}, \quad \varphi = \begin{pmatrix} Y \\ 0 \end{pmatrix}, \quad \tilde{\varphi} = \begin{pmatrix} \frac{\mu^2}{Y} \\ 0 \end{pmatrix}, \quad (\text{A.13})$$

Although the pseudo-moduli space (X, Y) is two-dimensional in this case, we can imagine working at fixed Y , and then the theory for X is still of the form (2.1). To see this explicitly, we expand around a point on (A.13),

$$\begin{aligned} \Phi &= \begin{pmatrix} \delta\Phi_{00} & \delta\Phi_{01} \\ \delta\Phi_{10} & X \end{pmatrix}, & \varphi &= \begin{pmatrix} Y + f_1(Y)\delta\chi_+ + f_2(Y)\delta\chi_- \\ \delta\varphi_1 \end{pmatrix}, \\ \tilde{\varphi} &= \begin{pmatrix} \frac{\mu^2}{Y} + f_2^*(Y)\delta\chi_+ - f_1(Y)\delta\chi_- \\ \delta\tilde{\varphi}_1 \end{pmatrix} \end{aligned} \quad (\text{A.14})$$

where $f_1(Y) = (1 + |Y|^4/\mu^4)^{-1/2}$ and $f_2(Y) = (1 + |Y|^4/\mu^4)^{-1/2}Y^2/\mu^2$. Then the superpotential becomes

$$W = \left(hX(\delta\varphi_1\tilde{\delta\varphi}_1 - \mu^2) + \frac{h\mu^2}{Y}\delta\Phi_{01}\delta\varphi_1 + hY\delta\Phi_{10}\delta\tilde{\varphi}_1 \right) + \left(\frac{h\sqrt{\mu^4 + |Y|^4}}{Y}\delta\Phi_{00}\delta\chi_+ \right) + (\text{cubic}) \quad (\text{A.15})$$

where (cubic) refers to terms that are cubic in the fluctuations. We see that to quadratic order, the model splits into two disconnected sectors — the first is an O’Raifeartaigh-type model of the form (2.1) and the second is supersymmetric and massive. At one-loop, the effective potential for X comes solely from the first sector. It has

$$M = \begin{pmatrix} 0 & 0 & \frac{h\mu^2}{Y} & 0 \\ 0 & 0 & 0 & hY \\ \frac{h\mu^2}{Y} & 0 & 0 & 0 \\ 0 & hY & 0 & 0 \end{pmatrix}, \quad N = \begin{pmatrix} 0 & h & 0 & 0 \\ h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (\text{A.16})$$

From this, we compute \mathcal{F} using (2.13) and substitute into (2.14) to find m_X^2 . We again find that the second term vanishes, while the first term is nonzero and gives the positive result

$$\begin{aligned} m_X^2 &= \frac{1}{16\pi^2 h^2 \mu^4} \int_0^\infty v^3 \left(\frac{4h^8 \mu^8}{(v^2 + h^2|Y|^2) \left(v^2 + \frac{h^2\mu^4}{|Y|^2} \right) \left(v^2 + h^2|Y|^2 + \frac{h^2\mu^4}{|Y|^2} \right)} \right) \\ &= \frac{h^4}{8\pi^2 |Y|^2 (|Y|^4 - \mu^4)} \left(|Y|^8 \log \frac{|Y|^4 + \mu^4}{|Y|^4} - \mu^8 \log \frac{|Y|^4 + \mu^4}{\mu^4} \right) \end{aligned} \quad (\text{A.17})$$

In the limit $|Y| \rightarrow \mu$ (which is where the potential for Y is minimized [11]), it reduces to

$$m_X^2 \rightarrow \frac{h^4 \mu^2 (\log 4 - 1)}{8\pi^2} \quad (\text{A.18})$$

which is precisely the answer obtained in [11].

References

- [1] L. O’Raifeartaigh, *Spontaneous symmetry breaking for chiral scalar superfields*, *Nucl. Phys. B* **96** (1975) 331.
- [2] M. Dine, J.L. Feng and E. Silverstein, *Retrofitting O’Raifeartaigh models with dynamical scales*, *Phys. Rev. D* **74** (2006) 095012 [[hep-th/0608159](#)].
- [3] M. Dine and J. Mason, *Gauge mediation in metastable vacua*, *Phys. Rev. D* **77** (2008) 016005 [[hep-ph/0611312](#)].
- [4] R. Kitano, H. Ooguri and Y. Ookouchi, *Direct mediation of meta-stable supersymmetry breaking*, *Phys. Rev. D* **75** (2007) 045022 [[hep-ph/0612139](#)].
- [5] H. Murayama and Y. Nomura, *Gauge mediation simplified*, *Phys. Rev. Lett.* **98** (2007) 151803 [[hep-ph/0612186](#)].
- [6] C. Csáki, Y. Shirman and J. Terning, *A simple model of low-scale direct gauge mediation*, *JHEP* **05** (2007) 099 [[hep-ph/0612241](#)].
- [7] O. Aharony and N. Seiberg, *Naturalized and simplified gauge mediation*, *JHEP* **02** (2007) 054 [[hep-ph/0612308](#)].
- [8] S.A. Abel and V.V. Khoze, *Metastable SUSY breaking within the standard model*, [hep-ph/0701069](#).
- [9] A. Amariti, L. Girardello and A. Mariotti, *On meta-stable SQCD with adjoint matter and gauge mediation*, *Fortschr. Phys.* **55** (2007) 627 [[hep-th/0701121](#)].
- [10] H. Murayama and Y. Nomura, *Simple scheme for gauge mediation*, *Phys. Rev. D* **75** (2007) 095011 [[hep-ph/0701231](#)].
- [11] K. Intriligator, N. Seiberg and D. Shih, *Dynamical SUSY breaking in meta-stable vacua*, *JHEP* **04** (2006) 021 [[hep-th/0602239](#)].
- [12] A.E. Nelson and N. Seiberg, *R symmetry breaking versus supersymmetry breaking*, *Nucl. Phys. B* **416** (1994) 46 [[hep-ph/9309299](#)].
- [13] K. Intriligator and N. Seiberg, *Lectures on supersymmetry breaking*, *Class. and Quant. Grav.* **24** (2007) S741 [[hep-ph/0702069](#)].
- [14] E. Witten, *Mass hierarchies in supersymmetric theories*, *Phys. Lett. B* **105** (1981) 267.
- [15] C.R. Nappi and B.A. Ovrut, *Supersymmetric extension of the $SU(3) \times SU(2) \times U(1)$ model*, *Phys. Lett. B* **113** (1982) 175.
- [16] K. Intriligator, N. Seiberg and D. Shih, *Supersymmetry breaking, R-symmetry breaking and meta-stable vacua*, to appear.
- [17] S. Ray, *Some properties of meta-stable supersymmetry-breaking vacua in Wess-Zumino models*, *Phys. Lett. B* **642** (2006) 137 [[hep-th/0607172](#)].
- [18] M. Claudson and M.B. Wise, *Effective potentials for a class of supersymmetric theories*, *Phys. Lett. B* **113** (1982) 31.
- [19] M. Dine and W. Fischler, *A phenomenological model of particle physics based on supersymmetry*, *Phys. Lett. B* **110** (1982) 227.
- [20] M. Dine and W. Fischler, *A supersymmetric GUT*, *Nucl. Phys. B* **204** (1982) 346.

- [21] L. Alvarez-Gaume, M. Claudson and M.B. Wise, *Low-energy supersymmetry*, *Nucl. Phys.* **B 207** (1982) 96.
- [22] S. Dimopoulos and S. Raby, *Geometric hierarchy*, *Nucl. Phys.* **B 219** (1983) 479.
- [23] T. Banks and V. Kaplunovsky, *Nosonomy of an upside down hierarchy model. 1*, *Nucl. Phys.* **B 211** (1983) 529.
- [24] V. Kaplunovsky, *Nosonomy of an upside down hierarchy model. 2*, *Nucl. Phys.* **B 233** (1984) 336.